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Golden Ratio Information for Neural Spike Code

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Abstract

Spike bursting is a ubiquitous feature of all neuronal systems. Assuming the spiking states form an alphabet for a communication system, what is the optimal information precessing rate? And what is the channel capacity? Here we demonstrate that the quaternary alphabet of spike number code gives the maximal processing rate, and that a binary source in Golden Ratio distribution gives rise to the channel capacity. A multi-time scaled neural circuit is shown to satisfy the hypotheses of this neural communication system.

Keywords: Neural spike code; Spike bursting; Neural communication system; Scaled neuron model

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Introduction

Claude E. Shannon's mathematical theory for communication laid a corner stone for the information age in the mid-20th century. In his model, there is an information source to produce messages in sequences of a source alphabet, an encoder/transmitter to translate the messages into signals in time-series of a channel alphabet, a channel to transmit the signals, and a receiver/decoder to convert the signals back to the original messages [1]. The mapping from the source alphabet to the channel alphabet is a source-to-channel encoding. Any communication system is characterized by two intrinsic performance parameters. One, its Mean Transmission Rate (MTR) (in bit per time) for all possible information sources, such as your Internet connection speed for all kinds of sources, both wanted and unwanted. Two, its Channel Capacity (CC) which a particular information source or a particular source-to-channel encoding scheme can take advantage to transmit at the fastest data rate.

One property that is ubiquitous to all neurons is their generation of electrical pulses across the cell membrane. The form can be in voltage or any type of ionic current. The form is not important to our discussion but only the state of excitation or quiescence. We further simplify our discussion by considering only those type of neurons or neuronal circuit models which are capable of spike bursts. A spike burst is a segment of time-series which both begins and ends with a quiescent phase but with a sequence of fast oscillations of equal amplitude and similar periods in between. In terms of dynamics, the quiescent phase or the refractory phase is slow, and the spiking phase is fast, typical of fast-slow, multi-timed neural models, c.f. [2-5]. The fast-slow distinction between the two phases and the spike number count allow us to transform the analogue wave form into a discrete information system as an obvious assumption, see also [6].

So, if we construct a communication system using one neuron as an encoder/transmitter and another as a receiver/decoder, then what would be the best MTR and CC? More aptly, in a network of neurons, any interneuron plays the role of transmission channel, the same questions arise. That is, how much information the neuron can process in a unit time when it is open for any information and when it is restricted to information of a particular type?

Materials and Methods

Measurements of communication system

Table 1 lists the definitions of essential parameters and variables for any information system [7]. Given a source whose messages are sequences of its source alphabet $S\ell = \{s_1, s_2, \ldots, s_\ell\}$, a source-to-channel encoding is a mapping from the source alphabet $S\ell$ to the set of finite sequences of the system alphabet A_n so that a source sequence in $S\ell$ is translated to a sequence in A_n. A particular source when source-to-channel coded in A_n is characterized by a probability distribution p, with p_k being the probability to find letter b_k at any position of the encoded messages. According to the information theory, letter or base b_k contains $log_2(1/p_k)$ bit of information for the encoded source, and a typical base on average contains $H(p)$bit of information for the encoded source. In statistical mechanics, the quantity entropy, $H(p)$ is a measure of the disorder of a system, and in information theory, it is the measure of diversity for the encoded source. It is a simple and basic fact that the maximal entropy is reached if and only if the probability distribution is the equi probability, $= 1/n,\ 1 \le k \le n$ and the maximal entropy is $H_n = log_2(n)$. From the designer's point view of a communication system, the system is not for a particular source with a particular encoded distribution, but rather for all possible sources. For example, the Internet is not designed for a particular source but rather for all sources, such as texts, videos, talk-show radios, and so on. In other words, it is only reasonable to assume that each base b_k will be used by some source, and it will be used equally likely when averaged over all sources. Hence, for any n-alphabet communication system, the maximal information entropy $H_n = log_2(n)$ in bit per base is a system-wise payoff measure for the alphabet.

This does not mean that the larger the alphabet size the better the system. The balance lies in the consideration of the payoff against cost. The primary cost is the times the system takes to process the alphabet, such as to represent or to transmit the bases. Assume a base b_k takes a fixed amount of time, τ_k , to process. Then for the equi probability distribution, the average processing time is $T_n(\tau) = \sum_k^n = 1\ \tau_k / n$ in time per base. Hence, the key performance parameter for an n-alphabet communication system is this payoff-to-cost ratio, $R_n(\tau) = H_n / T_n(\tau)$ in bit per time, referred to as the Mean Transmission Rate (MTR) or the transmission rate for short. This is an intrinsic measure for all communication systems, regardless of the size nor the nature of their alphabets. As a result, different systems can be objectively compared. In the example of Internet, the transmission rate is the measure we use to compare different means of connection, such as coaxial cable, optic fiber or satellite. Notice also that the mean transmission rate is determined by all sources, and it is in this sense that the transmission rate is a passive measurement of a communication system.

From the perspective of a particular source (user), its particular payoff-to-cost ratio is the source transmission rate, $R(p,\tau) = H(p)/T(p,\tau)$ with p being its distribution over the system alphabet A_n. This rate may be faster or slower than the mean rate $Rn(\tau)$. In other words, there is a potential gain for a particular source to exploit the system so that its bit rate $Rn(p,\tau)$ is no worse than the mean. The mathematical problem is to maximize the source transmission rate $Rn(p,\tau)$ over all choices of the distribution p. Solution to the optimization problem gives rise to the channel capacity (CC), denoted by K_n.

Table 1: Definitions for communication system parameters.

Variable average ± SD	Variable average ± SD
$A_n = \{b_1, b_2, ..., b_n\}$,	System alphabet for encoding, transmitting, and decoding.
$\tau = \{\tau_1, \tau_2, ..., \tau_n\}$	Base transmitting/processing time τk for base bk.
$p = \{p_1, p_2, ..., p_n\}$	Probability distribution over An for an encoded source
$H(p) = \sum_{k=1}^n P_k = 1\ p_k\ log_2(1/p_k)$	Averaged information entropy in bit per symbol of a particular source with encoded distribution p
$T(p,\tau) = \sum_{k=1}^n p_k \tau_k$	Averaged transmitting/processing time per symbol of a particular source with encoded distribution p.
$R(p,\tau) = H(p)/T(p,\tau)$	Particular transmission rate in bit per time of a particular source with encoded distribution p.
Hn = log2 n	Maximal entropy Hn = maxp{H(p)} with the equiprobability, pk = 1/n, 1 ≤ k ≤ n, for all sources.
$R_n(\tau) = H_n / \sum_{k=1}^n \tau_k / n$	Mean Transmission Rate (MTR) in bit per time for all sources with the equiprobability.
$K_n(\tau) = max_p \{R(p,\tau)\}$	Channel Capacity (CC) with $p_k = p_1^{\tau k/\tau 1}$, $\sum_{k=1}^n p_1^{\tau k/\tau 1} = 1$

Result for neural spike code

The number of spikes in a burst for a neuron is called the spike number and without introducing extra notation we denote it by b_k (as letters for the alphabet A_n with the subscript k ≥ 1 for the spike number of the burst. The resultant alphabet system is referred to as the neural spike code.

We consider first the simplest case when the process time for the code progresses like the natural number: $\tau_1 = a,\ \tau_2 = 2a, \ldots, \tau_k = ka$, for code base $b_1, b_2, \ldots, b_k$, respectively, for a fixed parameter a. That is, the 1-spike base takes 1 unit of time in a to process; the 2-spike base takes 2 units of time in $2a$ to process and so on. Without loss of generality we can drop a by assuming a =1 for now.

For the MTR, $R_n = H_n / [\ \Sigma_{k=1}^n\ k/n]$, of A_n, we have

$$R_n(\tau) = log_2\ n/(n+2)/2$$

Since $\Sigma_{k=1}^n k/n = n(n+1)/2/n$ For $n = 2,\ 3,\ \ldots, 12$, the values of R_n are 0.67, 0.79, 0.80, 0.77, 0.74, 0.70, 0.67, 0.63, 0.60, 0.58, 0.55

That is, the optimal spike code is A_4 with the MTR $R_4 =$ 0.80, as one can easily show the function $log_2\ x/(x+2)/2$ is a decreasing function for × > 4.

For the CC, $K_n(\tau)$, consider the binary spike case A_2 first.

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To simplify notation, we use $p = p_1$ and $q = p_2$, $p+q=1$. Then $H = -p\ log_2\ p - q\ log_2\ q,\ T = p\tau_1 + q\tau_2 = p + 2q$, and $R(p,\ q) = H / T$. We use the Lagrange multiplier method to the find the maximum $R(p,\ q)$ of subject to the constraint $g(p,\ q) = p + q = 1$. This is to solve the following system of equations:

$$R_p\ = \lambda g_p \quad \left[(-\ log_2\ p - 1/ln2)T - H\right]/T^2 = \lambda$$

$$R_p\ = \lambda g_p \Rightarrow \left[(-\ log_2\ q - 1/ln2)T - 2H\right]/T^2 = \lambda$$

$$g = 1 \quad p + q = 1$$

Equate the left-sides of the first two equations and simplify to get

$$T\ log_1\ p = T\ log_2\ q + H$$

implying that $log_2\ p - log_2\ q = \ H / T = R$. That is, at the maximal distribution $(p,\ q)$, the maximal rate K_2 is

$$R = H / T = log_2(p/q) \Longleftrightarrow log_2\ q = log_2\ p - H / T \quad \text{...........(1)}$$

To find $(p,\ q)$, we use the relation $q = 1 - p$ from the constraint $g(p,\ q) = 1$ and the identity above to rewrite $T = p + 2q = 2 - p$ and then to replace all q in H as follows

$$\begin{aligned} H &= -p\ log_2\ p - q\ log_2\ q \\ &= -p\ log_2\ p - (1-p)\left[log_2\ p - H/T\right] \\ &= -p\ log_2\ p - (1-p)log_2\ p + (1-p)H/T \\ &= -log_2\ p + (T-1)H/T \\ &= -log_2\ p + H - H/T \end{aligned}$$

which is simplified to

$$R = H / T = -log_2 p \quad \text{.............................(2)}$$

From Eq. (1) and Eq.(2) we have

$$-\ log_2\ p = log_2(p/q) \Longleftrightarrow 1/p = p/q \Longleftrightarrow (p\ +\ q)/p = p/q$$

The last equality shows p/q is the Golden Ratio with

$$p = \Phi = \frac{\sqrt{5}-1}{2} - 0.6180 \ and\ q = p^2 = \Phi^2$$

and the channel capacity is

$$K_2 = R = -log_2\Phi = 0.6943 > R_2 = 0.67$$

Better than the binary MTR.

In fact, this result is a special case of the following theorem which is a variation of Shannon's result from [1]. A proof is a straightforward generalization of the Golden Ratio case above.

Theorem 1: For an n-alphabet communication system, its source transmission rate $R(p,\tau) = H(p)/T(p,\tau)$ reaches a unique maximum $Kn(\tau) = max_p\{R(p,\tau)\}$ at an encoded source distribution p which is the solution to the following equations,

$$p_k = p^{\tau k/\tau 1} for\ 1 \le\ k \le n \sum_{k=1}^{n} p^{\tau k/\tau 1} = 1 \quad \text{.................... (3)}$$

and the maximal rate (the channel capacity) is $Kn(\tau) = -log_2\ p_1 / \tau_1$

Proof. (For review only.) Since the maximization is independent from the base presenting time τ , we will drop all references of it from the function T and R. The proof is based on the Lagrange multiplier method to maximize $R(p)$ subject to the constraint $g(p) = \Sigma_{k=1}^{n} p_k = 1$. This is to solve the joint equations: $\nabla R(p) = \lambda \nabla g(p),\ g(p) = 1$, where ∇ is the gradient operator with respect to p and λ is the Lagrange multiplier. Denote $R_{pk} = \partial R / \partial_{pk}$. Then the first system of equations becomes $R_{pk} = \left[H_{pk}\ T - H\tau_{pk}\right]/T2 = \lambda g_{pk} = \lambda$, component wise. Write out the partial derivatives of H and T and simplify, we have $-(log_2\ pk + 1/ln\ 2)T - H\tau_k = \lambda T$ for k=1, 2. . . n. Subtract the equation for k=1 from each of the remaining n−1 equations to eliminate the multiplier λ and to get a set of n−1 new equations:

$$-(log_2\ p_k - log_2\ p_1)T - H(\tau_k - \tau_1) = 0$$

which solves to

$$R = \left[log_2(p_k / p_1)\right]/(\tau_1 - \tau_k) = log_2\ \left[(p_k / p_1)^{1/(\tau 1 - \tau k)}\right]$$

Introducing a new quantity $\eta := 2^R = 2^{H/T}\ or\ H = T\ log_2\eta$ we can rewrite the equation above as $\eta = (pk / p1)^{1/(\tau 1 - \tau k)}$ and equivalently

$$pk = \eta^{\tau 1 - \tau k} p_1 \quad \text{............................ (4)}$$

for all k. Next we express the entropy H in terms of η and p1, τ1, substituting out all pk :

$$H = -\sum_{k=1}^{n} p_k\ log_2\ p_k = -\sum_{k=1}^{n}\ pk\left[(\tau_1 - \tau_k)\ log_2\ \eta + log_2\ p_1\right]$$

$$= -\left[\tau_1\ log_2\ \eta - \sum_{k=1}^{n} p_k\tau_k\ log_2\ \eta + log_2\ p_1\right] = -\left[\tau_1\ log_2\ \eta\ +\ log_2\ p_1\right] + T\ log_2\ \eta$$

Where we have used $\Sigma_{k=1}^{n}\ p_k = 1$ and $T = \Sigma_{k=1}^{n}\ p_k\tau_k$. Since we have by definition $H = T\ log_2\ \eta$, cancelling H from both sides of the equation above gives $log_2\ p_1 + \tau_1\ log_2\ \eta = 0$ and consequently

$$R = log_2\ \eta = log_2\ \left[p_1^{-1/\tau 1}\right] \text{ or } = p^{-1/\ 1} \quad \text{..............................(5)}$$

and from (4)

$$p_k = \eta^{\ \tau 1 - \tau k}\ p_1 = \ p^{\tau k/\tau 1} \quad \text{..............................(6)}$$

Last, solve the equation $f(p_1) = g(p) = \Sigma_{k=1}^{n}\ p_1^{\tau k/\tau 1} = 1$ for p1. Since f (p1) is strictly increasing in p1 and f (0)=0 <1 and f(1)=n > 1, there is a unique solution p1 ∈ (0, 1) so that f(p1)=1. By (5) and (6), the channel capacity is $K_n = R(p) = -[log_2\ p_1]/\tau_1 = -[log_2\ p_k]/\tau_k$. This completes the proof.

The Golden Ratio case is a corollary to the theorem with the assumption that $\tau_2 = 2\tau_1$ for n=2. And for the special case when $\tau_k = k\tau_1$ with n > 2, the CC distribution satisfies $pk = p_1^k$ for 1 ≤ k ≤ n with

$$p_1 + p^2{}_1 + p^3{}_1 + \cdots + p^n{}_1 = 1 \quad \text{....................(7)}$$

Denote the solution by $p_1 = \beta_n$ for A_n, then we can easily show that βn is a decreasing sequence in n, converging to 0.5 and bounded exactly from above by the Golden ratio $\beta_2=\Phi=0.6180$.

Multi-time scaled neuron model

In, a neuron model was discovered that remains symmetric under the conductance and resistance transformation: r=1/g and g=1/r [8]. The symmetry gives rise to the conductance characteristics for both ion channels and protein channels:

$$\Phi_X(V_X,\eta_X,Q_X) = heaviside\big(sign(\eta_X)(V_X-Q_X)\big)\, tanh^2\left[|\eta_X|(V_X-Q_X)/2\right] \quad \text{.............(8)}$$

The model looks exactly the same under the transformation gr=1 and ψ=1/ϕ, the resistance characteristics, leading to a unique model for any given neuron, rather than innumerable ad hoc models as conventionally is the case. This model then predicts that the phenomenon of spontaneous firing of individual ion channels [9,10] by ways of quantum tunneling [11] is both sufficient and necessary, marking the first transition from the quantum realm to the microscopic world in neuronal modeling. The model automatically gives rise to different time scales for ion and protein channels, permitting dramatic simplifications in dimensional reduction. The model also clearly lays out a blueprint for circuit implementation by the channel characteristics (8). The model is capable of both action potential propagation and spike-burst generation. Here we consider a four channel neuron model:

$$\begin{cases} CV' = -[\bar{g}_K\, n(V-E_k) + \bar{g}_{Na} m(V-E_{Na}) \\ +[\bar{g}_G\, h(V-E_G) + \bar{g}_{CX} c(V-E_{CX})] \\ n' = \alpha_K \sqrt{(n+\epsilon_K)/(\phi_K(V)+\epsilon_K)}\,(\phi_K(V)-n) \\ m' = \alpha_{Na}\sqrt{(m+\epsilon_{Na})/(\phi_{Na}(V)+\epsilon_{Na})}\,(\phi_{Na}(V)-m) \\ h' = \alpha_G \sqrt{(h+\epsilon_G)/(\phi_G(V)+\epsilon_G)}\,(\phi_G(V)-h) \\ c' = \sigma\alpha_{Cx}\sqrt{(c+\epsilon_{Cx})/(\phi_{Cx}(V)+\epsilon_{Cx})}\,(\phi_{Cx}(V)-c) \end{cases}$$

where Na is for the sodium channel, K is for the potassium channel, G is for the sodium-potassium gating channel, and the fourth channel, C_x, can be a calcium channel or a chlorine channel, or a protein channel, which are needed for spike-burst. Parameter σ takes only a fixed sign value, +1 or −1, because both can generate spike burst. Without the spontaneous firing parameters X, the model encounters a dividing-zero singularity and a flat-lining equilibrium, meaning the neuron are neither functional nor alive. All results outlined above are obtained in [8].

The rate parameters, α_X, naturally make the model multi-time scaled. For example, the full 5-dimensional system (9) can be reduced to a 3-dimensional system below by assuming the gating and the sodium channels to be the fastest with α_{Na}, α_G sufficiently large so that $m=\phi_{Na}(V)$ and $h=\phi_G(V)$:

$$\begin{cases} CV' = -[\bar{g}_K\, n(V-E_k) + \phi_{Na}(V)(V-E_{Na}) \\ +[\bar{g}_G\, \phi_G(V)(V-E_G) + \bar{g}_{CX} c(V-E_{CX})] \\ n' = \alpha_K \sqrt{(n+\epsilon_K)/(\phi_K(V)+\epsilon_K)}\,(\phi_K(V)-n) \\ c' = \sigma\alpha_{Cx}\sqrt{(c+\epsilon_{Cx})/(\phi_{Cx}(V)+\epsilon_{Cx})}\,(\phi_{Cx}(V)-c) \end{cases} \quad \text{.................(10)}$$

Simulations can be down on both with their dynamics indistinguishable for large α_{Na}, α_G. System (10) is another multi-time system, with the V -equation fast, the n-equation slow or comparable, and the c-equation slower. The lower dimensional reduction (10) can be advantageous for analytical manipulations c.f. [12].

Eq. (9): σ=−1, E_K=−60.0 mV, $\bar{g}_k$ =70 m. mho/cm², Q_K=−43.0 mV, $\bar{g}_{Na}$ =0.04/mV, E_{Na}=45.0 mV, =332.0 m. mho/cm², Q_{Na}=−52.0 mV, η_{Na}=0.01/mV, E_{Cx}=40.0 mV, $\bar{g}_{Cx}$ =20.0 m. mho/cm², Q_{Cx}=−50.0 mV, η_{Cx}=0.06/mV, E_G=−55.0 mV, $\bar{g}_G$ =8.0 2 m.mho/cm², Q_G=75 mV, η_G=0.03/mV, C=1μF/cm², αk=7.0/ms, and $\epsilon_k=\epsilon_{Na}=\epsilon_{Cx}=\epsilon_G=10^{-5}$. The changing parameter is for $1/\alpha_{CX}\in$ in ms (1, 110). Quantitatively similar result also holds (not shown) for the same parameters except for σ=+1, Q_{Cx}=−42.0 mV, η_{Cx}=1.0/mV, and α_{Cx} between 0.01/ms and 1/ms. All spike bursts start at the same initial values V (0)=−49, n(0)=m(0)=0, h(0)=1, and c(0)=0.025.

In **Figures 1A and 1B**, all parameters are fixed from except for the rate parameter α_{Cx} which is used as a bifurcation parameter. The plot is presented against $1/\alpha_{Cx}$ for a better visibility [8]. From the graph we can conclude immediately that if we denote the first bifurcation of k spikes by α_{Cx}, $_k$, then the sequence scales like the Harmonic sequence

$$\alpha_{Cx,\,k} \sim 1/k$$

and its renormalization

$$(\alpha_{Cx,\,k+1} - \alpha_{Cx,k})/(\alpha_{Cx},k - \alpha_{Cx,\,k-1}) \to 1 \;as\; k \to \infty$$

Converges to a universal number which is the first natural number 1, according to the neural spike renormalization theory of [13,14]. Second, the refractory time for the k-spike burst is proportional to the bursting time because their ratio is approximately a constant around 0.5, so that the kth spike base time τ_k for letter b_k is approximately (1+0.5) × bursting time for b_k . Thirdly, from the spike frequency plot we can conclude that it is approximately a constant around 1 cycle per msec for all spike bursts. As a result, the k-spike burst takes about k msec. All these values can be obtained by choosing an α_{Cx} value from the k-spike interval which is called the isospike interval. Hence, we can conclude empirically that **(Figures 2A and 2B)**

$$\tau_k = \tau_* \times k$$

For some constant τ_* around 1.5 msec. Hence, the hypothesis is satisfied for the information distribution equation (7) for the Golden Ratio distribution (n=2) and for the generalized Golden Ratio distribution (n > 2).

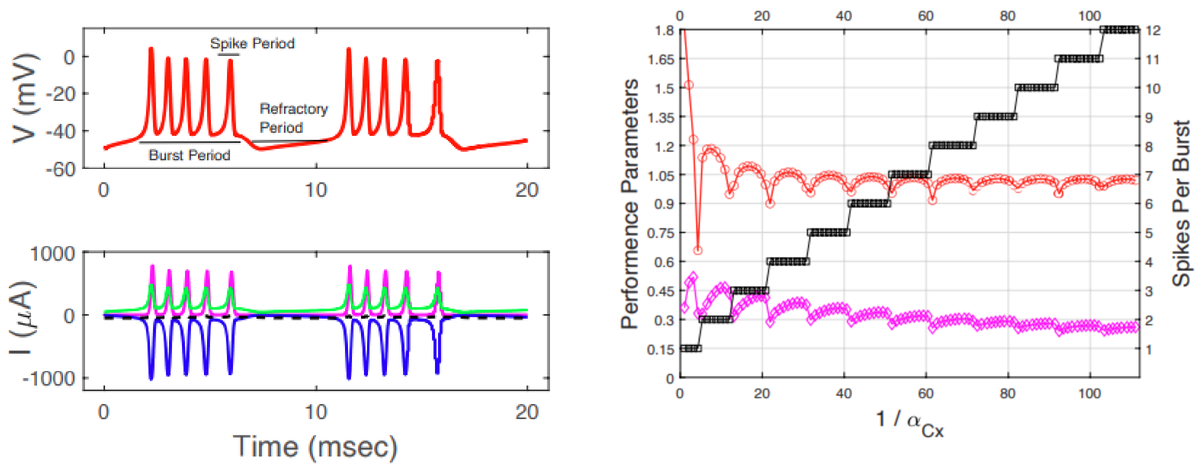

Figure 1 (A) Spike bursts and terminology Legend; **Note:** (▬) I_K; (▬) I_{Na}; (▬) I_G; (▬ ▬ ▬) I_{Ca} ; (B) Parameter values for the neural model. **Note:** (○) Spike Frequency; (◇) Refractory: Burst Period Ratio; (□) Spike Number.

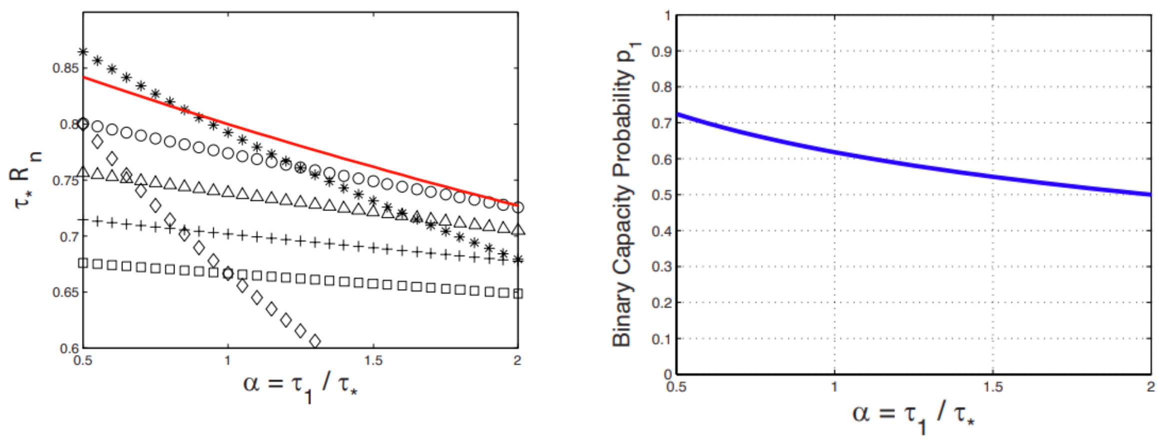

Figure 2 (A) Mean Rate Comparison. R_4 seems to be the likely optimal solution for most practical choices of α; (B) Channel capacity probability p1 for a binary information source. **Note:** (◇) n = 2; (✳) n = 3; (▬) n = 4; (○) n = 5; (△) n = 6; (+) n = 7; (□) n =8.

Results and Discussion

From **Figure 1A** we can see that the 1-spike burst is different from the rest. Although we can find parameter value from the 1-spike parameter interval to behave similarly to the rest of the spike bursts in the base process time τ_1 , but in practice, such a value can be hard to fixed. Instead, its spike-frequency can be higher or lower than the average of the rest. This is due to a phenomenon, referred to as spike frequency adaptation [4,5], for neural models. Let τ∗ be the same parameter as in (11) that is the average spike-burst period for k ≥ 2 and $\tau_k = \tau_* k \text{ for } k \geq 2$. For the 1-spike burst, we express its processing time as a scalar multiple of τ_* as $\tau_1 = \alpha\tau_*$ for some α either greater, or equal to, or smaller than 1. Thus, for the MTR, R_n , for the A_n code,

$$T_n = \sum_{k=1}^{n} \tau_k / n = \tau_* \left((\alpha-1)/n + (n+1)/2\right)$$ and Rn is

$$R_n(\tau) = H_n / T_n(\tau) = (\log_2 n) / \tau_* \left((\alpha-1)/n + (n+1)/2\right)$$

Figure 2A plots its graph against the parameter α. It shows for the range of 0.5 < α < 2, the optimal MRT is with A3 or A4. For the binary alphabet's channel capacity, the optimal distribution for the 1-spike burst is computed numerically from the equation (3):

$$p_1 + p_1^{\tau 2/\tau 1} = p_1 + p_1^{2/\alpha} = 1$$

for each α from the interval (1/110, 1). **Figure 2B** shows the graph of the solution as a function of α. It goes through the Golden Ratio at α=1 as expected.

Obviously, nature has built a communication system out of neurons. The results above suggest that such systems come with inherent preferences in information. There are many examples of memory related animal systems where the number of preferred modes for operation is around 4 the so-called magic number 4 phenomena, c.f. [15-18]. The underlining hypothesis is that animal brains have a neurological tendency to maximize information entropy against time in cost. Information sources with the Golden Ratio distribution are many. One example is the Golden Sequence, 101101011011010110101..., which is generated by starting with a symbol 1 and iterating the sequence according to 2 rules: replace every symbol 1 in the sequence by 10 and replace every symbol 0 in the sequence by 1. The distribution {p0, p1} of the symbols {0, 1} along the sequence has the Golden Ratio distribution: $p_0 + p_1 = 1, p_0 / p_1 = p_1 / 1 = \Phi$. This gives a perfect illustration of Shannon's fundamental theory of noiseless channel; assigning symbol 1 to the 1-spike burst base and symbol 0 to the 2-spike burst base for the source-to-channel encoding gives rise to the binary neural system's channel capacity. Penrose's aperiodic tiling is another example with Golden Ratio distribution. In its simplest form, its bases consist of a 54-degree rhombus and a 72-degree rhombus. The frequencies with which the rhombi appear in the plane follow the Golden Ratio distribution [19]. Again, a trivial but most natural source-to-channel encoding gives rise to the fastest source transmission rate for the binary neural code.

Equation (12) for α gives a way to quantify how close something is to the Golden Ratio. For example, a rectangular frame is uniquely defined by its height-to-width (aspect) ratio. A frame of the Golden Ratio is 1:1.6180=1:1/Φ. To translate it into a statistical distribution over a binary

source, both height and width need to be proportionated against the height-width sum. Thus, the width: sum fraction is $p_1 = 1/\Phi / \left(1 + 1/\Phi\right) = 1/ \left(\Phi + 1\right) = \Phi$, the Golden Ratio, which in turn corresponds to α=1 if {p1, 1−p1} is the binary channel capacity distribution. The aspect ratio of a typical wide-screen monitor is 10:16, corresponding to a p1=16/26=0.6154 binary distribution, and α ~ 0.99 for the neural binary channel capacity. The aspect ratio of a high definition TV is 9:16, corresponding to a p1=16/25=0.64 binary distribution, or α ~ 0.90. All fall inside the R4-optimal mean rate range and near the Golden Ratio distribution for binary channel capacities as shown in **Figure 2A**.

Conclusion

In conclusion, with the assumption that the number of spikes per burst forms a letter of an alphabet and the processing times for the letters progress like the natural number, then the neural spike code with the first four letters achieves the best average information rate, and for any binary source, the information distribution in Golden Ratio achieves the binary channel capacity for the neural spike code. Altogether, our result seems to support the hypothesis that human's brain is biologically build for informational preferences.

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Declarations

Ethical approval

Not Applied.

Competing interests

None.

Authors' contributions

Not Applied.

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None.

Availability of data and materials

All data generated or analysed during this study are included in this published article. Matlab mfiles are available from the corresponding author on reasonable request.

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